Radiative Energy-Loss of Heavy Quarks in a Quark-Gluon Plasma

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We estimate the radiative energy-loss of heavy quarks, produced from the initial fusion of partons, while propagating in a quark-gluon plasma which may be formed in the wake of relativistic heavy ion collisions. We find that the radiative energy-loss for heavy quarks is larger than the collisional energy-loss for all energies. We point out the consequences on possible signals of the quark-gluon plasma.

One of the most interesting predictions of QCD is the transition from the confined/chirally broken phase to the deconfined/chirally symmetric state of quasi-free quarks and gluons, the so-called quark-gluon plasma (QGP). Relativistic heavy ion collisions are being studied with the intention of investigating the properties of the QGP [1]. While experiments at AGS and SPS continue, new experiments have been planned at RHIC and LHC with centre of mass energies 200 AGeV and 5.5 ATeV, respectively. During the past decade many different signatures of the transition to the QGP have been proposed. A promising example is the emission of penetrating probes such as dileptons and single photons, which can reveal the early parton dynamics and the history of evolution of the plasma. Similarly, the production and propagation of open charm and high energy jets in a dense medium, can provide information [2] about parton scattering and thermalization of the partonic system. Jets are expected to show up at collider energies at RHIC and LHC.

Heavy quark pairs are mostly produced from the initial fusion of partons (mostly from $gg \to Q\bar{Q}$, but also from $q\bar{q} \to Q\bar{Q}$, where q denotes one of the lighter quarks and Q is a heavy quark) of the colliding nucleons and also from the QGP, if the initial temperature is high enough, which is likely to be achieved at RHIC and LHC energies. The charm quarks will be produced on a time scale of $1/2m_c \simeq 0.07$ fm/c, which would be as low as $\simeq 0.02$ fm/c for bottom quarks. There is no production of heavy quarks at late times in the QGP and none in the hadronic matter. Thus, the total number of heavy quarks gets frozen very early in the history of the collision which makes them a good candidate for a probe of the QGP. Immediately upon their production, these heavy quarks will propagate through the deconfined matter and start losing energy. The energy-loss suffered by these quarks will determine the shape of the dilepton spectra produced from correlated charm (or bottom) decay which provides a large background to dilepton production from annihilation of quarks in the plasma. We shall come back to this aspect towards the end of this Letter.

There are two contributions to the energy-loss of a heavy quark in the QGP: one caused by elastic collisions with the light partons of the QGP and the other by radiation of the decelerated color charge, *i.e.*, bremsstrahlung of gluons. There is an extensive body of literature [3–8] on the collisional energy-loss of energetic quarks considering elastic collisions with the quarks and gluons ($Qg \to Qg$ and $Qq \to Qq$) of the dense medium. A complete leading order result for the collisional energy-loss of heavy quarks has been found using the hard thermal loop resummation technique [9].

It is well known that the contribution of the radiative processes ($Qq \to Qqg$ and $Qg \to Qgg$) is of the same order in the coupling constant as the collisional energy-loss [9]. The estimate of the radiative energy-loss in the past has been discussed by a number of authors [10–13] within perturbative QCD taking into account the Landau-Pomeranchuk suppression due to multiple collisions. These studies, however, were limited to the case of massless energetic quarks and gluons. As far as we know, there is no estimate of the radiative energy-loss for heavy quarks in the literature. It is not easy to extend the sophisticated treatment of multiple scattering formulated by the authors of Ref. [13] to the case of heavy quarks.

Until such a detailed investigation is performed, an extension of the work of Ref. [10] for the radiative energy-loss of massless quarks to the case of heavy quarks can provide valuable insight. This approach is similar to the one by Gyulassy, Wang and Plümer [12] and leads to almost identical results in the case of light partons. Baier et al. [13], on the other hand, found a different dependence of the radiative energy-loss on the energy of the parton by including the rescattering of the emitted gluon in the QGP. However, inserting typical values for the energy of the parton, the temperature, and the coupling constant yields quantitatively similar results. Therefore we will restrict ourselves to the simple approach of Ref. [10] in

the present work and discuss the consequences of our estimate for signatures of the QGP.

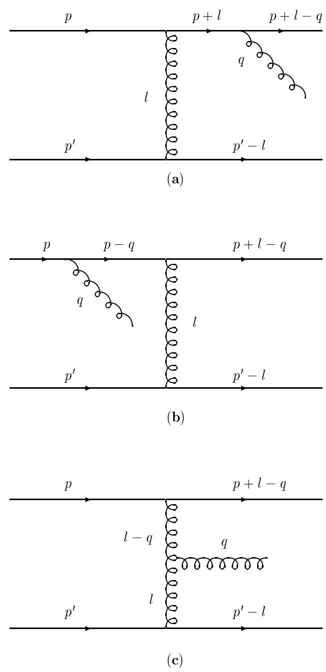

FIG. 1. Feynmann diagrams for gluon bremsstrahlung from quarks.

We start from an expression for the gluon emission probability which has been derived by Gunion and Bertsch [14] in the case of light partons assuming a factorization of the matrix elements of Fig. 1 into elastic scattering and gluon emission. It can be shown that their result also holds if one of the light quarks in Fig. 1 is replaced by a heavy quark assuming that the engery of the emitted gluon is not too large ($q_0 \ll \sqrt{s}$). The final result for the multiplicity distribution of the radiated gluon can then be written as

$$\frac{dn_g}{d\eta d^2 q_\perp} = \frac{C_A \alpha_s}{\pi^2} \frac{l_\perp^2}{q_\perp^2 \left(\vec{q}_\perp - \vec{l}_\perp\right)^2} \quad , \tag{1}$$

where $q = (q_0, \vec{q}_\perp, q_3)$ and $l = (l_0, \vec{l}_\perp, l_3)$ are the four momenta of the emitted and the exchanged gluons, respectively, and $\eta = (1/2)\ln[(q_0 + q_3)/(q_0 - q_3)]$ is the rapidity. $C_A = 3$ is the Casimir invariant of the adjoint representation. The factorization in (1) was obtained in the limit $x\, l_\perp << q_\perp$, where x is the fractional momentum carried by the radiated gluon relative to the maximum available, such that the radiation is confined to a uniform (central) rapidity region, the most important zone in relativistic heavy ion collisions. Eq.(1) holds also for gluon bremsstrahlung emitted by a gluon.

Now we can estimate the radiative energy-loss per unit length for heavy quarks by multiplying the interaction rate Γ and the average energy-loss per collision ν, which is given by the average of the probability of radiating a gluon times the energy of the gluon. One can further correct for the Landau-Pomeranchuk effect by including a formation time restriction [10,11] through a step function $\theta(\tau - \tau_f)$. This puts a restriction on the phase space of the emitted gluons in which the formation time, τ_f must be smaller than the interaction time, $\tau = 1/\Gamma$. The formation time is estimated by requiring the separation between the emitted gluon and the parton from which it is emitted to be $r_\perp = v_\perp t > 1/q_\perp$ ($q_\perp \equiv |\vec{q}_\perp|$) according to the uncertainty principle. Using $v_\perp = q_\perp/q_0$ and $q_0 = q_\perp \cosh\eta$, we find $\tau_f = \cosh\eta/q_\perp$.

The average radiative energy-loss per collision is calculated as

$$\nu = \langle\ n_g q_0\ \rangle = \int d\eta d^2 q_\perp \frac{dn_g}{d\eta d^2 q_\perp}\ q_0\ \theta\left(\tau - \tau_f\right)\ . \tag{2}$$

Performing the integration in (2) in the limit $(q_\perp \tau)^2 >> 1$ and $q_\perp >> l_\perp$, we get

$$\nu \simeq \frac{6\alpha_s}{\pi} \langle\ l_\perp^2\ \rangle \tau \ln\left(\frac{q_\perp^{\rm max}}{q_\perp^{\rm min}}\right)\ . \tag{3}$$

For the infrared cut-off $q_\perp^{\rm min}$ we choose the Debye screening mass of a pure gluon gas,

$$q_\perp^{\rm min} = \mu_D = \sqrt{4\pi\alpha_s}\, T\ , \tag{4}$$

where T is the temperature of the system. For heavy quarks of mass M, the square of the maximum transverse momentum of the emitted gluon is given by

$$(q_\perp^{\rm max})^2 = \left\langle \frac{(s - M^2)^2}{4s} \right\rangle\ , \tag{5}$$

where s is the Mandelstam variable. To evaluate (5) we need to compute $\langle s\rangle$ and $\langle 1/s\rangle$ leading to

$$\begin{aligned} \langle s\rangle \ &= \ M^2 \ + \ 2p'E\ , \\ \langle 1/s\rangle \ &= \ \frac{1}{4p'p} \ln\left[\frac{M^2 + 2Ep' + 2pp'}{M^2 + 2Ep' - 2pp'}\right]\ , \end{aligned} \tag{6}$$

where E and p are the energy and momentum of a incoming heavy quark and p' is the average momentum of the light quark or gluon of the QGP. The average value of p' can be taken as $\sim 3T$. Now, (5) becomes,

$$(q_\perp^{\rm max})^2 = \frac{3ET}{2} - \frac{M^2}{4} + \frac{M^4}{48pT} \ln\left[\frac{M^2 + 6ET + 6pT}{M^2 + 6ET - 6pT}\right]. \tag{7}$$

The average momentum transfer of the scattering process is defined as

$$\langle l_\perp^2\rangle \simeq \langle l^2\rangle \equiv \langle t\rangle = \frac{\int_{\mu_D^2}^{q_\perp^{{\rm max}^2}} dt\ t\ d\sigma/dt}{\int_{\mu_D^2}^{q_\perp^{{\rm max}^2}} dt\ d\sigma/dt}\ , \tag{8}$$

where the differential cross section for elastic scattering is

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\,(s-M^2)^2}|\mathcal{M}|^2 \ , \tag{9}$$

and the square of the matrix element, $|\mathcal{M}|^2$ can be obtained from Ref. [15]. In the limit $|t| << s$, the differential cross section in (9) can be approximated by

$$\frac{d\sigma}{dt} \sim \frac{1}{t^2} \quad . \tag{10}$$

We have checked that the modification of the energy-loss using the full expression for $|\mathcal{M}|^2$ is negligible (see below). Combining (8) to (10) we get

$$\langle l_\perp^2 \rangle \simeq \frac{\mu_D^2\,(q_\perp^{\max})^2}{(q_\perp^{\max})^2 - \mu_D^2}\ \ln\left[\frac{(q_\perp^{\max})^2}{\mu_D^2}\right] \quad . \tag{11}$$

The radiative energy-loss for heavy quarks is then obtained by combining (3) and (11) and multiplying by $\Gamma = 1/\tau$,

$$\left(-\frac{dE}{dx}\right)_{\rm rad} = \frac{3\alpha_s}{\pi}\frac{\mu_D^2\,(q_\perp^{\max})^2}{(q_\perp^{\max})^2-\mu_D^2}\ln^2\left[\frac{(q_\perp^{\max})^2}{\mu_D^2}\right] . \tag{12}$$

Since the mass of the quark in this expressions enters only via the maximum transverse momentum (7) the radiative energy-loss of a heavy quark differs from the one of a massless quark only for small energies of the order of M.

Let us also recall the expression for the collisional energy-loss of heavy quarks considered in Ref. [9] using the hard thermal loop resummation technique. In the domain $E << M^2/T$, it reads

$$\begin{aligned}\left(-\frac{dE}{dx}\right)_{\rm coll.} &= \frac{8\pi\alpha_s^2T^2}{3}\left(1+\frac{n_f}{6}\right)\left[\frac{1}{v}-\frac{1-v^2}{2v^2}\right. \\ &\times \left.\ln\left(\frac{1+v}{1-v}\right)\right]\ln\left[2^{\frac{n_f}{6+n_f}}B(v)\frac{ET}{m_gM}\right],\end{aligned} \tag{13}$$

whereas for $E >> M^2/T$, it is

$$\begin{aligned}\left(-\frac{dE}{dx}\right)_{\rm coll.} &= \frac{8\pi\alpha_s^2T^2}{3}\left(1+\frac{n_f}{6}\right) \\ &\times \ln\left[2^{\frac{n_f}{2(6+n_f)}}\,0.92\frac{\sqrt{ET}}{m_g}\right],\end{aligned} \tag{14}$$

where v is the velocity of the heavy quarks, $B(v)$ is a smooth function of v, which can be taken approximately as 0.7, n_f is the number of light quark flavours taken as 2.5, and $m_g = \sqrt{(1+n_f/6)/3}\,g_sT$ the thermal gluon mass.

It should be noted that the collisional and the radiative energy-loss are of the same order in the coupling constant [9], although the latter is caused by higher order diagrams within naive perturbation theory. The reason for this behaviour is the fact that the interaction rate entering into the radiative energy-loss suffers from a quadratically infrared singularity using a bare propagator for the exchanged gluon, whereas this divergence is reduced to a logarithmic one for the collisional energy-loss. This reduction is caused by the presence of the energy transfer of the exchanged gluon in the definition of the collisional energy-loss [9]. In the case of the radiative energy-loss, on the other hand, this factor is absent, because the energy-loss is caused by the emitted and not by the exchanged gluon. Using a hard thermal loop resummed propagator the quadratic singularity in the interaction rate is reduced to a logarithmic one and the final result is of higher order ($\Gamma \sim \alpha_s$) than naively expected. Multiplying this rate by the gluon emission probability (1) yields the result (12) of order α_s^2. Using a resummed propagator for the collisional energy-loss leads to the finite expressions (13) and (14) of the same order.

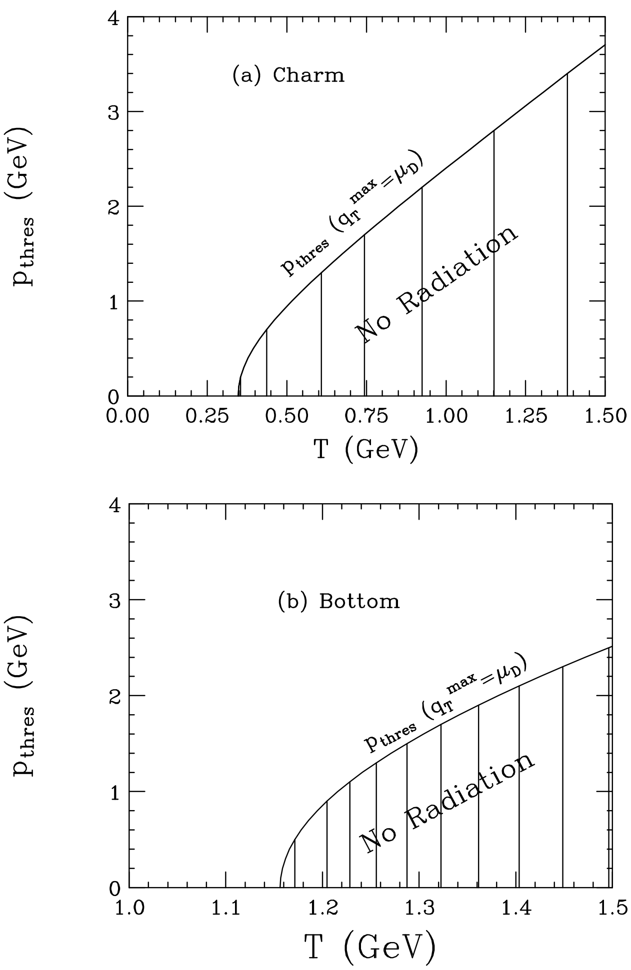

FIG. 2. The threshold momenta of the heavy quark as a function of T.

The expression for the radiative energy-loss in (12) ex-

hibits a threshold behaviour: for $(q_\perp^{\max})^2 < \mu_D^2$ there is no radiative energy-loss which is shown by the hatched area in Fig. 2. (Obviously, this behaviour will be different for mass-less quarks.) We see that the value of the threshold momenta of the heavy quarks, below which there is no radiation, increases with increasing temperature.

In Fig. 3, we compare our results with that of the collisional energy-loss for heavy quarks obtained in Ref. [9] as a function of energy at a temperature $T = 500$ MeV and α_s =0.3 for charm and bottom quarks. Before discussing our results, we give the justification for the approximation made in (10) for the differential cross section used in computing the radiative energy-loss, which enabled us to obtain the closed form given above. The solid lines represent the radiative energy-loss of heavy quarks with full $|\mathcal{M}|^2$ whereas the dashed lines correspond to that with the approximate expression. We see that retaining only the $\sim 1/t^2$ term in $d\sigma/dt$ is sufficient for our purpose. The dash-dotted lines in Fig. 3 represent the collisional energy-loss obtained in Ref. [9]. We find that the radiative energy-loss dominates over the collisional one at all energies. For $E > 20$ GeV the difference amounts to an order of magnitude.

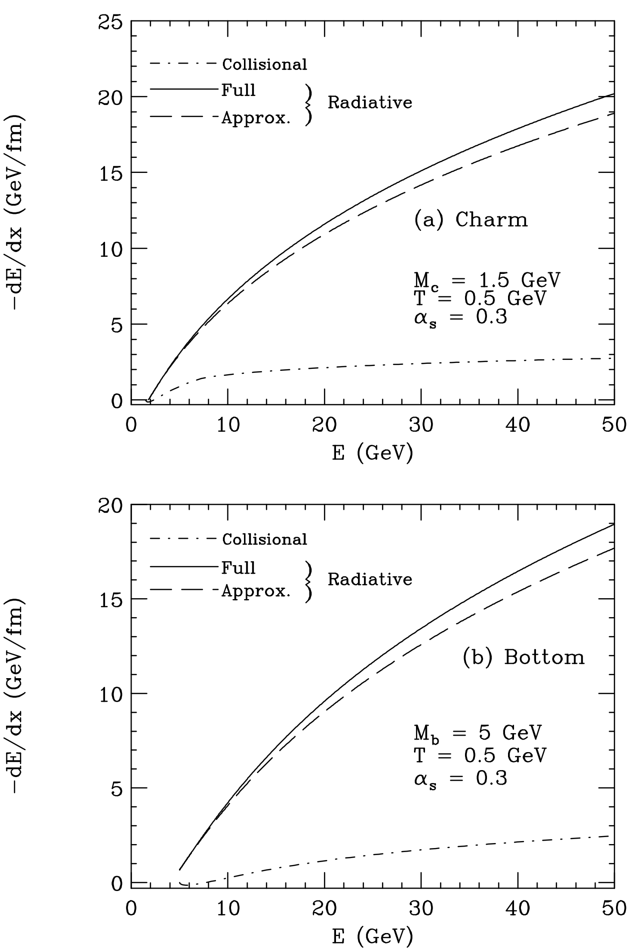

FIG. 3. The energy-loss of heavy quarks as a function of their energy.

The QGP expected to be produced at RHIC and LHC is likely to be far from chemical equilibrium, initially. Chemical reactions among the partons will then push it towards a chemical equilibrium [16–18]. The evolution of the temperature and the quark and gluon fugacities[1] at RHIC and LHC eneriges has recently been obtained [18] for such a scenario with initial conditions from a Self Screened Parton Cascade Model [19]. As a first estimate, the expressions for the collisional energy-loss given earlier (Eqs. (13), (14)) can be modified by replacing the terms $(1 + n_f/6)$ by $(\lambda_g + \lambda_q n_f/6)$ and $n_f/(6 + n_f)$ by $\lambda_q n_f/(6\lambda_g + \lambda_q n_f)$ to account for the departure from chemical equilibrium. Alternatively one may use the results of Ref. [20] for a non-equilibrium plasma. The radiative energy-loss is modified by using the non-equilibrium Debye mass [17]

$$\mu_D^2 = 4\pi\lambda_g\alpha_s T^2. \tag{15}$$

in (12).

This has interesting consequences. It has been shown recently [20] that considering *only* the collisional energy-loss, in this manner, amounts to having only a small drag on the motion of heavy quarks in such a plasma, at least at RHIC energies, where the charm quarks were found to loose only $\sim$ 10% of their energy during their propagation, through the plasma and upto 40% of their initial energy at LHC, in a collision involving two gold nuclei [21].

The drag acting on the heavy quark is conveniently defined by writing

$$-\frac{dE}{dx} = Ap, \tag{16}$$

where A denotes a drag-coefficient in the spirit of the treatment used earlier in literature [4,20] and p is the momentum of the heavy quark. Adding the collisional and radiative energy-loss experienced by a heavy quark in such an equilibrating and cooling plasma, we have found that

$$A \simeq C/\tau, \tag{17}$$

where C is a slowly varying function of p with $C \simeq 0.4$ for charm quarks at RHIC energies and ~ 0.7 at LHC energies, for $E \leq 5 - 6$ GeV. This leads to a rather large drag of ~ 1.6/fm at RHIC and ~ 2.7/fm at LHC on charm quarks at $\tau = \tau_i$, where $\tau_i = 0.25$ fm/c is determined by the onset of the kinetic equilibrium [18].

This has a very important implication. Consider a charm quark having an energy E_i of the order of a few GeV at time τ_i in such an expanding and chemically equilibrating plasma. Due to this large drag, the charm quark

[1]The fugacities are defined [17] as $f_i = \lambda_i \tilde{f}_i$, where f is the distribution of the partons and $\tilde{f}$ is the corresponding equilibrium distribution.

produced initially will come to rest very quickly and diffuse. We have verified this by performing numerical calculations of the final momentum of charm quarks which propagate under such a drag. We find that, irrespectively of the initial energy ($E_i \leq$ 5–6 GeV) the final energy of the charm quark is about 1.5 – 1.6 GeV, in the cases considered here. Recall again that the charm quarks do not come to a stop if only the collisional energy-loss is included [20].

Thus we conclude that the radiative energy-loss of heavy quarks produced initially in relavisitic nuclear collisions plays a dominant role in pulling them to a stop in the QGP both at RHIC and LHC energies. Their final momentum distribution will then be determined by the temperature at which the hadronization takes place. This could be the temperature of the mixed phase, if such a phenomenon takes place.

We may add that Svetitsky and co-workers [4] have actively investigated such a scenario. In their work the large drag coefficient arises due to a large value of $\alpha_s \simeq 0.6$ and a fully equilibrated plasma, even though only the collisional energy-loss is included.

Let us return to the discussion of the momentum distribution of charm quarks. (Similar considerations hold for bottom quarks.) It is expected that the momentum distribution of charm quarks will be reflected in the momentum distribution of charmed mesons, whose correlated decay will provide a back-ground to dileptons from quark annihilation. We see immediately that a look at the $p_\perp$ distribution of these leptons may help us to isolate the two contributions, as they should be very different for the two sources. Shuryak [22] and Lin et al. [23] have argued that the correlated charm decay back-ground for dileptons may be suppressed if the energy-loss of charm quarks is taken as 1–2 GeV/fm. Our study lends a strong support to their conclusion which were obtained by attributing an arbitrarily assumed value for the energy-loss.

In conclusion, we have estimated the radiative energy-loss of heavy quarks propagating in a quark gluon plasma. This, along with the (fairly small) collisional energy-loss acts as a strong drag force on heavy quarks, which pulls them to a stop even in a chemically equilibrating and cooling plasma. This ensures that the momenta of the resulting charm mesons will be determined by the hadronization temperature. The correlated decay of such charm mesons will then no longer pose a back-ground for dileptons having their origin in the quark-antiquark annihilation at least at large invariant mass. This separation could even be made easier by measuring the $p_\perp$ distribtuion of the lepton pairs. In a future publication we shall report the result of the transverse hydrodynamic flow of the plasma on these conclusions [18].

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